$$= -\nabla \times (\nabla \phi) = 0$$

$$\boxed{\nabla \times q = 0}$$

Which is the necessary and sufficient condition for the motion to be irrotational

If $\nabla \times q \neq 0$ Rotational

—o—

Ex-23 / 52 $\quad q = \dfrac{A(xj - yi)}{x^2+y^2} \qquad A = \text{constant}$

$q = iu + jv$

To Prove

1) The motion is a Possible motion for an incompressible fluid.
2) To obtain the equation to streamlines
3) Motion is of Potential kind.
4) Determine the velocity Potential.

Sol.1) Satisfied $\quad u = -\dfrac{Ay}{x^2+y^2}$

$$v = \frac{Ax}{x^2+y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(\frac{Ay}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{Ax}{x^2+y^2}\right) = 0$$

$$\Rightarrow \frac{2Axy}{(x^2+y^2)^2} - \frac{2Axy}{(x^2+y^2)^2} = 0$$

$$0 = 0$$

Which is true Hence it is a Possible liquid motion.

—o—

II. Steady and irrotational flow:- If the motion is steady then $\frac{\partial \phi}{\partial t} = 0$

$$\Rightarrow \int \frac{dp}{\rho} + \frac{1}{2} q^2 + \Omega = \text{constant}$$

If the fluid is homogeneous and incompressible then variation of density remain constant then

$$\boxed{\frac{p}{\rho} + \frac{1}{2} q^2 + \Omega = \text{constant}}$$

This is known as Bernoulli's equation for steady and irrotational flow.

—x—

Euler's equations of motion:-

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = X - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = Y - \frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = Z - \frac{\partial p}{\partial z}$$

—x—

Conservation of Momentum

The momentum of a body is defined as the Product of the mass of the body and its velocity

i.e. velocity $= \frac{mq}{g_0}$

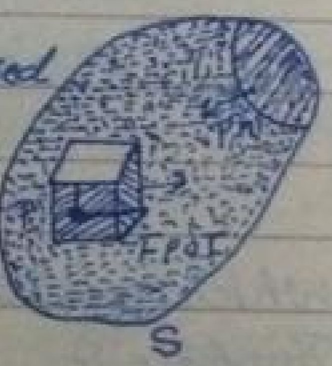
Has the dimensions of force-time In the flow of fluids the momentum M Per unit Volume is given By

$$M = \frac{\sigma q}{g_0} = \rho q$$

the velocity is a Vector quantity so Momentum is likewise a Vector quantity having magnitude and Both direction.

Equation of motion of an inviscid fluid:-

Consider any arbitrary closed surface S drawn in the region occupied by the incompressible fluid at an instant t.



ρ be the density of the fluid Particle at the Point P. with the closed surface And $d\tau$ we the volume of the fluid enclosing be the Point P.

Mass of the fluid element $= \rho \, d\tau$

$= $ volume $\times$ density

$$\boxed{x = \frac{x_0}{t_0} t, \; y = y_0 e^{t-t_0}, \; z = z_0} \quad \text{—— (1)}$$

Hence the Path lines are given by
Let the fluid Particle x_0, y_0, z_0 Passing Through a fixed Point at (x_1, y_1, z_1) along an instant of time t
$= T$ such that $t_0 \leq T \leq t$

$$x_1 = \frac{x_0}{t_0} T$$

$$y_1 = y_0 e^{T-t_0} \quad \text{—— (2)}$$

$$z_1 = z_0$$

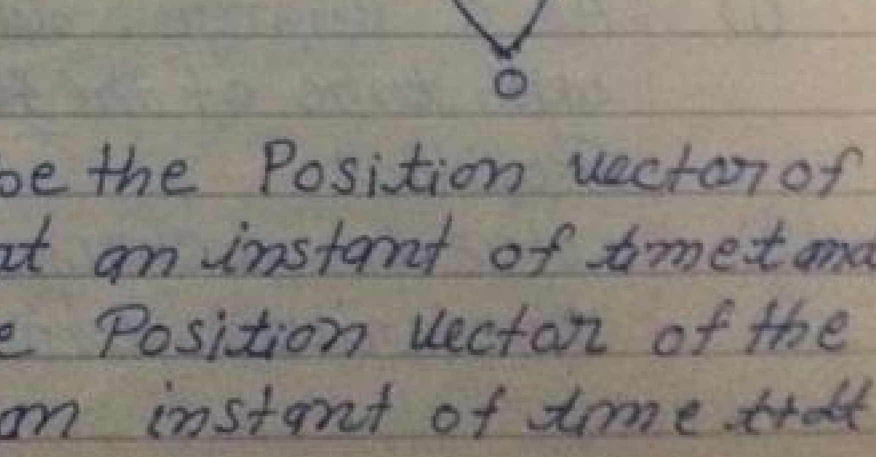
Where T is the Parameter
From (1) And (2) we have

$$\boxed{x = \frac{x_1}{T} t, \; y = y_1 e^{t-T}, \; z = z_1}$$

This is known as Eq^n. to streaklines

Combination, Local, Convective and Material derivatives:-

$\frac{ds}{dt} dt = 2dt, \; \frac{dr}{dt} dt = 2dt$



Consider r be the Position vector of the Point P at an instant of time t and $r + dr$ be the Position vector of the Point Q at an instant of time $t + dt$

Each term represent a rate for the differen-tial element of volume.
consider a fluid element of infinitesimal volume δV and density ρ which is situated at a point r at any time t
mass of the fluid $= \rho \delta V$
Material derivative of the mass vanishes

$$\frac{D}{Dt}(\rho V) = 0$$

this is Eq^n of continuity in the simplest form
consider a closed surface S in a fluid medium containing a volume V fixed in space.
Let n is the unit outward drawn normal at a surface element δs. Let q is the fluid velocity at the surface element δs.
Then the normal component of the velocity q measured out ward will be
Rate of mass flow across $\delta s = \rho (n q) \delta s$
Total rate of mass flow $= \int_S \rho (n \cdot q) ds$

Total rate of mass flow in to $V = -\int_S \rho (n q) ds$
Using Gauss theorem, we have

$$= -\int_V \nabla (\rho q) dV$$

Total mass with in $V = \rho dV$